Math 214 - Fall 2019 - Quiz 4

From the totality of chapters we've read in the text, we have encountered "The Fundamental Theorem of Linear Algebra". Over the last few chapters, it has grown to include many statements which are all equivalent. Indeed, this theorem starts with the claim: **The following are equivalent:**, leading some to refer to this as a TFAE theorem! Below, list as many of these equivalent statements as possible.

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Math 214 - Fall 2019 - Quiz 4 - Solutions

From the totality of chapters we've read in the text, we have encountered "The Fundamental Theorem of Linear Algebra". Over the last few chapters (*), it has grown to include many statements which are all equivalent. Indeed, this theorem starts with the claim: **The following are equivalent:**, leading some to refer to this as a TFAE theorem! Below, list as many of these equivalent statements as possible.

1. A is invertible.
2. $Ax = b$ has a unique solution for every b in R^n.
3. $Ax = 0$ has only the trivial solution.
4. The reduced row echelon form of A, $RREF(A)$, is equal to I_n.
5. A is the product of elementary matrices.
6. $\text{rank}(A) = n$.
7. $\text{nullity}(A) = 0$.
8. The column vectors of A are linearly independent.
9. The column vectors of A form a basis for R^n.
10. The column vectors of A span R^n.
11. The row vectors of A are linearly independent.
12. The row vectors of A form a basis for R^n.
13. The row vectors of A span R^n.
14. $\det(A) \neq 0$.
15. 0 is not an eigenvalue of A.

(*) Linear Algebra, a Modern Introduction, 4th edition.
Author: David Poole, 2015, Cengage Learning. ISBN-13: 978-1-285-46324-7.