Math 214 - Fall 2019 - Quiz 2

1. Consider the matrix $A = \begin{bmatrix} 1 & -4 & 3 \\ 0 & 6 & -7 \\ 3 & 0 & 8 \end{bmatrix}$

   (a) Compute the determinant, $det(A)$, using appropriate elementary row operations:

   (b) Compute the determinant, $det(A)$, using the cofactor method and a wisely chosen row or column:

2. Suppose A and B are both 5 by 5 matrices, and that $det(A) = 4, det(B) = \frac{2}{3}$. Then compute the determinants of the following matrices, if possible, or state that this is not possible:

   (a) AB^2

   (b) $B^{-1}A$

   (c) $A^2 + B$

Math 214 - Fall 2019 - Quiz 2 - Solutions

1. Consider the matrix $A = \begin{bmatrix} 1 & -4 & 3 \\ 0 & 6 & -7 \\ 3 & 0 & 8 \end{bmatrix}$

   (a) Compute the determinant, $det(A)$, using appropriate elementary row operations:

   $$A = \begin{bmatrix} 1 & -4 & 3 \\ 0 & 6 & -7 \\ 3 & 0 & 8 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -4 & 3 \\ 0 & 6 & -7 \\ 0 & 12 & -1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -4 & 3 \\ 0 & 6 & -7 \\ 0 & 0 & 13 \end{bmatrix}$$

   Now that our row reduced form is that of a triangular matrix, our determinant is the product of entries along the main diagonal: $\mathbf{det(A) = 1 \cdot 6 \cdot 13 = 78}$

   (b) Compute the determinant, $det(A)$, using the cofactor method and a wisely chosen row or column:

   Choosing Row 1, because it contains a zero, we see:

   $$det(A) = 1 \cdot (-1)^{(1+1)} det(\begin{bmatrix} 6 & 7 \\ 0 & 8 \end{bmatrix}) + 3 \cdot (-1)^{(3+1)} det(\begin{bmatrix} -4 & 3 \\ 6 & -7 \end{bmatrix})$$

   $$= 48 + 3 \cdot 10 = \mathbf{78}$$

2. Suppose A and B are both 5 by 5 matrices, and that $det(A) = 4, det(B) = \frac{2}{3}$. Then compute the determinants of the following matrices, if possible, or state that this is not possible:

   (a) AB^2

   $det(AB^2) = det(A)det(B)^2 = 4 \cdot (\frac{2}{3})^2 = \mathbf{\frac{16}{9}}$

   (b) $B^{-1}A$

   $det(B^{-1}A) = \frac{1}{det(B)} \cdot det(A) = \frac{3}{2} \cdot 4 = \mathbf{6}$

   (c) $A^2 + B$

   It is **not possible** to determine this determinant, since we have no theorems regarding the determinant of the SUM of matrices!