Math 214 - Fall 2019 - Quiz 1

1. Let $\bar{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 2 \end{bmatrix}, \quad \bar{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 5 \end{bmatrix}, \quad \bar{v}_3 = \begin{bmatrix} 2 \\ 3 \\ 0 \\ 0 \end{bmatrix}$

   Is the set $V = \{\bar{v}_1, \bar{v}_2, \bar{v}_3\}$ a linearly independent set? Prove your answer.

2. Let $\bar{w}_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}, \quad \bar{w}_2 = \begin{bmatrix} -2 \\ 6 \end{bmatrix}, \quad \bar{p} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

   Is $\bar{p}$ a member of $\text{Span}(\{\bar{w}_1, \bar{w}_2\})$? Explain.

Math 214 - Fall 2019 - Quiz 1 - Solutions

1. Let $\bar{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 2 \end{bmatrix}, \quad \bar{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 5 \end{bmatrix}, \quad \bar{v}_3 = \begin{bmatrix} 2 \\ 3 \\ 0 \\ 0 \end{bmatrix}$

   Is the set $V = \{\bar{v}_1, \bar{v}_2, \bar{v}_3\}$ a linearly independent set? Prove your answer.

   The easiest way to answer this is to write these as row vectors, then produce the RREF matrix:

   $$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 5 \\ 2 & 3 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 5 \\ 0 & 3 & 0 & -4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & -19 \end{bmatrix}$$

   Since the rank of the RREF matrix is the same as the number of equations, (no rows became zero rows), V is a linearly independent set of vectors.

2. Let $\bar{w}_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}, \quad \bar{w}_2 = \begin{bmatrix} -2 \\ 6 \end{bmatrix}, \quad \bar{p} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

   Is $\bar{p}$ a member of Span($\{\bar{w}_1, \bar{w}_2\}$)? Explain.

   We see if there are any coefficients, c_1, c_2 for which $c_1\bar{w}_1 + c_1\bar{w}_1 = \bar{p}$.

   $$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ -3 & 6 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 0 & 4 \end{array}\right]$$

   Since we have an inconsistent system, there is NO solution set, c_1, c_2. So NO linear combination of $\bar{w}_1, \bar{w}_2$ can produce the vector $\bar{p}$, so we conclude that $\bar{p}$ is NOT an element of Span($\{\bar{w}_1, \bar{w}_2\}$).